4.3 Putting It All Together

You have used equations to model a variety of quadratic functions. You may have noticed some common characteristics of these equations. You have also observed patterns in the graphs and tables of quadratic functions.

To understand a relationship, it helps to look at how the value of one variable changes each time the value of the other variable increases by a fixed amount. For a linear relationship, the y-value increases by a constant amount each time the x-value increases by 1.

Look at this table for the linear relationship $y = 3x + 1$. The "first differences" are the differences between consecutive y-values.

Because the y-value increases by 3 each time the x-value increases by 1, the first differences for $y = 3x + 1$ are all 3.

Now, you'll look at differences for quadratic relationships.

$y = 3x + 1$

x	y	First differences
0	1	
1	4	$4 - 1 = 3$
2	7	$7 - 4 = 3$
3	10	$10 - 7 = 3$
4	13	$13 - 10 = 3$
5	16	$16 - 13 = 3$

Getting Ready for Problem 4.3

The simplest quadratic relationship is $y = x^2$, and it is the rule for generating square numbers. In fact, the word *quadratic* comes from the Latin word for "square."

The table below shows that the first differences for $y = x^2$ are not constant.

$y = x^2$

x	y	First differences
0	0	
1	1	$1 - 0 = 1$
2	4	$4 - 1 = 3$
3	9	$9 - 4 = 5$
4	16	$16 - 9 = 7$
5	25	$25 - 16 = 9$

- What happens when you look at the "second differences" for $y = x^2$?

$y = x^2$

x	y
0	0
1	1
2	4
3	9
4	16
5	25

First differences

$1 - 0 = 1$

$4 - 1 = 3$

$9 - 4 = 5$

$16 - 9 = 7$

$25 - 16 = 9$

Second differences

$3 - 1 = 2$

$5 - 3 = 2$

$7 - 5 = 2$

$9 - 7 = 2$

- Study the pattern of first and second differences for $y = x^2$. Do you think the tables for other quadratic functions will show a similar pattern?

Problem 4.3 Functions and Patterns of Change

A. 1. Make a table of values for each quadratic equation below. Include integer values of x from -5 to 5. Show the first and second differences as is done for the table above.

a. $y = 2x(x + 3)$ **b.** $y = 3x - x^2$

c. $y = (x - 2)^2$ **d.** $y = x^2 + 5x + 6$

2. Consider the patterns of change in the values of y and in the first and second differences. In what ways are the patterns similar for the four tables? In what ways are they different?

3. What patterns of change seem to occur for quadratic relationships?

B. 1. Make a table of (x, y) values for each equation below. Show the first and second differences.

a. $y = x + 2$ **b.** $y = 2x$ **c.** $y = 2^x$ **d.** $y = x^2$

2. Consider the patterns of change in the values of y and in the first and second differences. How are the patterns similar in all four tables? How are they different?

3. How can you use the patterns of change in tables to identify the type of relationship?

ACE **Homework starts on page 64.**